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Can Indeterminacy and Self-Fulfilling Expectations Help Explain International Business Cycles? A Preliminary Investigation

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Abstract

We introduce equilibrium indeterminacy into a two-country incomplete asset model with imperfect competition and analyze whether self-fulfilling, belief-driven fluctuations (i.e., sunspot shocks) can help resolve the major puzzles of international business cycles. We find that a combination of productivity and sunspot shocks can account for the observed counter-cyclical behavior in international relative prices and quantities, while simultaneously generating volatilities that match the data. The indeterminacy model can also resolve the Backus-Smith puzzle without requiring a low value of the trade elasticity.

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1 Introduction

Business cycle statistics suggest that the terms of trade and net exports are both countercyclical and volatile over the cycle, and the real exchange rate is negatively correlated with relative consumption.[1] Standard models, where productivity shocks drive international business cycles, struggle to replicate these key properties of the data.[2] This is partially due to the counterfactual international transmission mechanism implied by these models. In response to a positive domestic technology shock, the increase in domestic output is associated with a fall in relative prices (i.e. a depreciation of the terms of trade), whereas in the data positive output changes are associated with relative price increases. In this paper we show that equilibrium indeterminacy, which allows for self-fulfilling expectations or sunspot shocks, can help account for the observed fluctuations in relative international prices and cross-country trade flows.[3]

The model economy we consider is a two-country incomplete asset economy with imperfect competition. In each country, final consumption and investment goods are produced using domestic and foreign intermediate goods. Prices are assumed to be flexible and the real exchange rate deviates from purchasing power parity due to home bias towards domestically-produced intermediate goods. Indeterminacy is introduced via increasing returns to scale technology where the marginal cost schedule of intermediate firms is decreasing in output. By assuming variable capacity utilization, the model can generate indeterminacy for empirically plausible values for the steady state markup.

Our main findings are summarized as follows. We show that the empirical irregularities with the data can be resolved under indeterminacy using a combination of sunspot shocks and productivity shocks. International business cycle fluctuations driven solely by self-fulfilling expectations cannot replicate the main features of the data, since we find that the international transmission of sunspot shocks are qualitatively similar to productivity shocks. A sunspot-induced increase in domestic output is associated with a depreciation of the terms of trade and the so-called Backus-Smith puzzle emerges, since with domestic consumption higher than its foreign counterpart, the real exchange rate depreciates. The

[1] See, e.g., Backus and Smith (1993), Chari et al. (2002), Corsetti et al. (2008a), Benigno and Thoenissen (2008), Raffo (2008, 2010), and Engel and Wang (2011).

[2] See Raffo (2010) for an excellent summary of the international business cycle literature.

[3] By indeterminacy we mean that there exists multiple equilibrium paths which converge to the steady state.

empirical success of the indeterminacy model when belief shocks and productivity shocks enter the model is due to an unconventional transmission mechanism for productivity shocks that originate in the foreign country. With indeterminacy, goods market clearing is ensured via changes in foreign output, rather than via adjustments in the terms of trade. Consequently, the propagation mechanism of productivity shocks is no longer symmetric. While positive productivity shocks in the home country result in an expansion of the domestic economy, positive foreign productivity shocks have a temporary contractionary effect on foreign output and employment. This helps magnify adjustments in the terms of trade and the resulting expenditure switching effect towards home goods leads to increased volatility of real net exports and counter-cyclical responses. The Backus-Smith puzzle is also resolved since the depreciation in terms of trade generated by a positive home productivity shock can be offset by the combination of a negative foreign productivity shock and a negative sunspot shock. Thus, as relative consumption increases, the real exchange rate now appreciates.

This paper is related to a small literature that has investigated the role of indeterminacy in explaining business cycle fluctuations. This approach has yielded some success in explaining closed-economy business cycles (see, e.g., Benhabib and Farmer, 1994; Farmer and Guo, 1994; Schmitt-Grohe, 2000; Benhabib and Wen, 2004; Jaimovich, 2007). Similar to this paper, the studies of Guo and Sturzenegger (1998) and Xiao (2004) have also investigated indeterminacy as a possible explanation for international business cycles. However, both these studies are conducted using one-good models and their focus of attention rests solely on explaining the consumption-output anomaly. We show that indeterminacy can help explain a number of other important features of the data relating to relative international prices and cross-country trade flows.

This paper also contributes to the international real business cycle (IRBC) literature. The recent literature has attempted to resolve the above anomalies with varying degrees of success. One strand of the literature (e.g. Stockman and Tesar, 1995; Benigno and Thoenissen, 2008) has highlighted the importance of a non-traded goods sector in resolving the Backus-Smith puzzle. Another strand of the IRBC literature (e.g. Boileau, 1999; Engel and Wang, 2011) has emphasized the importance of traded capital goods in increasing the volatility of net exports and the terms of trade. A third strand (e.g. Heathcote and Perri, 2002; Corsetti et al., 2008a, 2008b; Thoenissen, 2010) has emphasized the role of the trade price elasticity parameter. If this parameter is sufficiently low, this generates a downward-sloping world demand for domestic traded goods (with respect to the terms of trade), and the terms of trade appreciates when domestic production expands.[4] Without being reliant upon a negative international transmission for productivity shocks, our results highlight that the indeterminacy model can perform better than the IRBC literature in matching the data. The indeterminacy model can generate countercyclical behavior for the terms of trade and real net exports, which are sufficiently volatile over the business cycle. This improvement in volatility is not at the cost of reduced volatility of the other aggregate variables relative to output, whose volatilities also either increased or remain unchanged. Furthermore, the Backus-Smith puzzle does not arise as the model now generates a negative correlation between the real exchange rate and the relative consumption. As is shown, this negative correlation remains regardless of the choice of trade price elasticities.

Finally, this paper is also related to the recent studies by Raffo (2010) and Karabarbounis (2014) who also aim to explain the anomalies in relative international prices and cross-country trade movements via the inclusion of additional sources of fluctuations. Karabarbounis (2014) introduces a labor wedge into an otherwise standard IRBC model with complete asset markets, whereas Raffo (2010) considers an additional source of technological variation: investment-specific technology shocks. This paper complements these two studies by offering an alternative explanation based on indeterminacy and endogenous fluctuations.

The remainder of the paper is organized as follows. Section 2 outlines the model economy. Section 3 discusses the calibration of the model. Section 4 compares the results for both the determinacy and indeterminacy scenarios. Finally, Section 5 briefly concludes.

2 Model

We develop a two-country extension of the imperfect competition model studied by Benhabib and Farmer (1994) and Schmitt-Grohe (1997) for the closed economy. Following Wen (1998), we assume variable capacity utilization which significantly reduces the size of the steady state mark-up needed to generate indeterminacy. Within each country there exists a

[4] Thoenissen (2010) shows that a conventional IRBC model can generate enough volatility for the terms of trade and avoid the Backus-Smith puzzle under a negative international transmission calibration. The range of values for the trade elasticity θ that generate these model properties is very narrow: $0.4113 \leq \theta \leq 0.4678$.

representative agent, two final good producers and a continuum of intermediate good producing firms. Intermediate firms operate under monopolistic competition and use domestic labor and capital as inputs to produce tradeable goods. The competitive final good producers use domestic and imported intermediate goods to produce non-tradeable consumption or investment goods, which are subsequently purchased by the domestic agent. However, final good producers are assumed to have a bias for domestically produced intermediate goods. While the law of one price is assumed to hold for all intermediate goods, with *home bias*, the real exchange rate deviates from purchasing power parity. The following presents the features of the model for the Home country on the understanding that the Foreign case can be analogously derived. All Foreign variables are denoted by an asterisk.

2.1 Final good producers

In each country, there are two final goods, consumption and investment, which are produced with homogenous of degree one production functions, using the intermediate goods as the only inputs. The Home consumption final good (C) is produced by a competitive firm that uses C_H and C_F as inputs according to the following CES aggregation technology index:

$$C_t = \left[a^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where the constant elasticity of substitution between aggregate Home and Foreign intermediate goods is $\theta > 0$ and the relative share of domestic and imported intermediate inputs used in the production process is $0 < a < 1$. The Home investment final good (I) is produced according to the following CES aggregation technology index:

$$I_t = \left[b^{\frac{1}{\rho}} I_{H,t}^{\frac{\rho-1}{\rho}} + (1-b)^{\frac{1}{\rho}} I_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{2}$$

where $\rho > 0$ and $0 < b < 1$. The inputs C_H, C_F, I_H and I_F are defined as the quantity indices of domestic and imported intermediate goods respectively:

$$C_{H,t} = \left[\int_0^1 c_t(j)^{\frac{\kappa-1}{\kappa}} dj \right]^{\frac{\kappa}{\kappa-1}}, \qquad C_{F,t} = \left[\int_0^1 c_t(j^*)^{\frac{\kappa-1}{\kappa}} dj^* \right]^{\frac{\kappa}{\kappa-1}},$$
$$I_{H,t} = \left[\int_0^1 i_t(j)^{\frac{\kappa-1}{\kappa}} dj \right]^{\frac{\kappa}{\kappa-1}}, \qquad I_{F,t} = \left[\int_0^1 i_t(j^*)^{\frac{\kappa-1}{\kappa}} dj^* \right]^{\frac{\kappa}{\kappa-1}},$$

where the elasticity of substitution across domestic (imported) intermediate goods is $\kappa > 1$, and $c(j)$, $c(j^*)$, $i(j)$, $i(j^*)$ are the respective quantities of the domestic and imported type j and j^* intermediate goods. Intermediate firms sell their products to both final consumption and investment final good producers, where it is assumed that the law of one price holds:

$$p_t(j) = S_t p_t^*(j), \qquad p_t(j^*) = S_t p_t^*(j^*),$$

where S_t is the nominal exchange rate. Cost minimization in final good production yields the demand conditions for Home and Foreign goods:

$$C_{H,t} = a \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t, \qquad C_{F,t} = (1-a) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t, \tag{3}$$

$$I_{H,t} = b \left(\frac{P_{H,t}^I}{P_t^I} \right)^{-\rho} I_t, \qquad I_{F,t} = (1-b) \left(\frac{P_{F,t}^I}{P_t^I} \right)^{-\rho} I_t, \tag{4}$$

and the corresponding aggregate price indices are given by:

$$P_t = \left[a P_{H,t}^{1-\theta} + (1-a) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad P_t^I = \left[b (P_{H,t}^I)^{1-\rho} + (1-b)(P_{F,t}^I)^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{5}$$

where P is the consumer price index, P^I is the price of investment goods and P_H, P_F, P_H^I, P_F^I are the respective price indices of Home and Foreign intermediate goods.

2.2 Intermediate goods producers

All intermediate firms have access to the same technology. A Home firm of type j has a production technology given by

$$Y_t(j) = Z_t \left(u_t(j) K_t(j) \right)^{\alpha} L_t(j)^{\gamma} - \phi, \quad j \in [0, 1] \tag{6}$$

where K and L represent capital and labor usage respectively, Z_t is the exogenous level of technology or productivity, and the input share is $\alpha + \gamma \geq 1$. The rate of capacity utilization $u_t \in (0, 1)$ is endogenously determined. Following Greenwood et al. (1988) it is assumed that the depreciation rate of capital δ_t is higher if it is used more intensively:

$$\delta_t = \frac{1}{\eta} u_t^{\eta}, \tag{7}$$

where $\eta > 1$. In addition, we also introduce a fixed cost of production $\phi > 0$. Therefore, regardless of how much output is produced, a proportion ϕ of the intermediate good is used up in each period. As in Schmitt-Grohe (1997), allowing for a fixed production cost enables the model to generate zero profits without imposing any restrictions on the size of the steady state markup.[5] Given competitive prices of labor and capital, cost-minimization yields:

$$w_t = \gamma mc_t(j) Z_t \left(u_t(j) K_t(j)\right)^{\alpha} L_t(j)^{\gamma-1}, \tag{8}$$

$$rr_t + \delta_t = \alpha mc_t(j) Z_t u_t^{\alpha}(j) K_t(j)^{\alpha-1} L_t(j)^{\gamma}, \tag{9}$$

$$u_t^{\eta} = \alpha mc_t(j) Z_t u_t^{\alpha}(j) K_t(j)^{\alpha-1} L_t(j)^{\gamma}, \tag{10}$$

where mc_t is real marginal cost and w_t and $(rr_t{+}\delta_t)$ are the respective real wage and user cost of capital.

Given that the total demand for firm j's output can be expressed as:

$$Y_t(j) = \left(\frac{p_t(j)}{P_{H,t}}\right)^{-\kappa} \left[C_{H,t} + C_{H,t}^{*}\right] + \left(\frac{p_t(j)}{P_{H,t}^{I}}\right)^{-\kappa} \left[I_{H,t} + I_{H,t}^{*}\right],$$

it follows from the firm's profit maximization problem that the optimal price-setting rule is:

$$p_t(j) = \chi mc_t(j) P_t, \tag{11}$$

where $\chi \equiv \frac{\kappa}{\kappa-1}$ is the mark-up.

2.3 Representative agent

The representative agent has an expected utility function of the form:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right)$$

[5] As discussed by Rotemberg and Woodford (1996), Schmitt-Grohe (1997), and Jaimovich (2007), positive profits are not observed in the US economy despite the presence of market power.

where C_t and L_t are consumption and work effort, and the discount factor is $0 < \beta < 1$. Following Greenwood *et. al* (1988) we assume that the period utility function is given by:

$$U(C_t, L_t) = \frac{1}{1-\sigma}\left[\left(C_t - \frac{\psi}{1+\nu}L_t^{1+\nu}\right)^{1-\sigma} - 1\right],$$

where $\sigma > 0$ is the relative risk aversion in consumption, $\nu \geq 0$ is the inverse of the Frisch elasticity of labor supply, and $\psi > 0$.

The agent during period t supplies labor and capital to the intermediate good producing firms, receiving real income from wages w_t, a rental return on capital rr_t and nominal profits from the ownership of domestic intermediate firms Π_t. The agent then uses these resources to purchase the two final goods, dividing purchases between consumption C_t and investment I_t. The purchase of an investment good forms next period's capital according to the law of motion

$$K_{t+1} = (1-\delta_t)K_t + I_t. \tag{12}$$

The asset market structure is assumed to be incomplete. The Foreign agent is able to trade two non-state contingent bonds, $B^*_{H,t}$ and $B^*_{F,t}$, whereas the Home agent can only purchase domestic bonds denoted by $B_{H,t}$. All bonds are denominated in units of the domestic aggregate consumption index. For the Foreign agent, there is a transaction cost Ψ of adjusting the internationally traded bond $B^*_{H,t}$, where it is assumed that Ψ is a positive and differentiable function.[6] This transaction cost is paid to financial firms and captures the costs of adjusting bond holdings and is sufficient to ensure that bond holdings are stationary.[7] Consequently, in real terms the period budget constraints of the Home and Foreign agent can be expressed as:

$$\frac{B_{H,t}}{r_t}+C_t+\frac{P_t^I}{P_t}I_t \leq B_{H,t-1}+\int_0^1 w_t L_t(j)dj+\int_0^1 (rr_t+\delta_t(j))K_t(j)dj+\int_0^1 \Pi_t(j)dj+R_t, \tag{13}$$

$$\begin{aligned}\frac{B^*_{H,t}}{Q_t r_t}\frac{1}{\Psi(B^*_{H,t})} + \frac{B^*_{F,t}}{r^*_t} + C^*_t + \frac{P_t^{*I}}{P^*_t}I^*_t \leq \frac{B^*_{H,t-1}}{Q_t} + B^*_{F,t-1} + \int_0^1 w^*_t L^*_t(j^*)dj^* \\ + \int_0^1 (rr^*_t + \delta^*_t(j^*))K^*_t(j^*)dj^* + \int_0^1 \Pi^*_t(j^*)dj^* + R^*_t,\end{aligned} \tag{14}$$

[6] Following Benigno (2001), we assume that $\Psi = 1$ when bond holdings are at their steady state level and Ψ is positive, differentiable, and strictly decreasing in a neighborhood of the steady state.

[7] For an in-depth discussion of the stationary problem of incomplete market, open-economy models, see Schmitt-Grohe and Uribe (2003) and Ghironi (2006).

where R_t and R_t^* denote rebates from financial firms, r_t and r_t^* denotes the Home and Foreign (gross) real interest rates, and $Q_t \equiv \frac{S_t P_t^*}{P_t}$ denotes the the CPI-based real exchange rate.

The Home agent's maximization problem yields:

$$U_c(C_t, L_t) = \left(C_t - \frac{\psi L_t^{1+\nu}}{1+\nu}\right)^{-\sigma} = \lambda_t \tag{15}$$

$$-\frac{U_L(C_t, L_t)}{U_c(C_t, L_t)} = \psi L_t^{\nu} = w_t \tag{16}$$

$$\lambda_t \frac{P_t^I}{P_t} = \beta E_t \lambda_{t+1} \left[rr_{t+1} + \delta_{t+1} + (1 - \delta_{t+1}) \frac{P_{t+1}^I}{P_{t+1}} \right] \tag{17}$$

$$\beta r_t E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] = 1, \tag{18}$$

where λ_t denotes the shadow price of wealth. Analogous conditions to (15)-(18) apply for the Foreign agent, where the interest rate parity condition can be derived to yield:

$$r_t = \frac{r_t^*}{\Psi(B_{H,t}^*)} E_t \left[\frac{Q_{t+1}}{Q_t} \right]. \tag{19}$$

Optimizing behavior implies that the budget constraints (13) and (14) hold with equality in each period and the appropriate transversality conditions are satisfied.

2.4 Market clearing and equilibrium

We now focus on a symmetric equilibrium where all firms in Home and Foreign set the same price in each period t, rent the same amount of capital, and employ the same amount of labor. Consequently, $p_t(j) = P_{H,t} = P_{H,t}^I$ and the index j can be dropped from all variables. Market clearing requires that

$$Y_t = C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^*, \tag{20}$$

and assuming that the Foreign non-state contingent bond is in zero net supply, bond markets clearing requires that:

$$B_{H,t} + B_{H,t}^* = 0 \quad B_{F,t}^* = 0. \tag{21}$$

The aggregate resource constraint is given by:[8]

$$C_t + \frac{P_t^I}{P_t} I_t + \frac{B_{H,t}}{r_t} = B_{H,t-1} + \frac{P_{H,t}}{P_t} Y_t. \tag{22}$$

From the price indices (5) and their Foreign equivalents, the following relative prices can be derived:

$$\frac{P_t^I}{P_t} = \frac{\left[b + (1-b) T_t^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[a + (1-a) T_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}, \tag{23}$$

$$\frac{P_{H,t}}{P_t} = \left[a + (1-a) T_t^{1-\theta}\right]^{\frac{1}{\theta-1}}, \tag{24}$$

$$T_t = \frac{\left[a + (1-a) T_t^{\theta-1}\right]^{\frac{1}{\theta-1}}}{\left[a + (1-a) T_t^{1-\theta}\right]^{\frac{1}{\theta-1}}} Q_t, \tag{25}$$

where T_t is the terms of trade.[9] In what follows, we call an increase (decrease) in the terms of trade, or the real exchange rate, a depreciation (appreciation). Finally, we measure net exports as the difference between exports and imports, divided by total output (all evaluated at steady state prices):[10]

$$NX_t = \frac{C_{H,t}^* + I_{H,t}^* - \overline{T}\left(C_{F,t} + I_{F,t}\right)}{C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^*}. \tag{26}$$

Equilibrium. An equilibrium for the world economy consists of a set of real prices r_t, r_t^*, w_t, w_t^*, rr_t, rr_t^*, δ_t, δ_t^*, mc_t, mc_t^*, λ_t, λ_t^*; a set of relative prices $\frac{P_{H,t}}{P_t}$, $\frac{P_{F,t}^*}{P_t^*}$, $\frac{P_t^I}{P_t}$, $\frac{P_t^{*I}}{P_t^*}$, Q_t, T_t; a collection of allocations for the Home and Foreign agent C_t, C_t^*, I_t, I_t^*, L_t, L_t^*, K_t, K_t^*, u_t, u_t^*, $B_{H,t}$, $B_{H,t}^*$, $B_{F,t}^*$; and a collection of allocations for Home and Foreign final and intermediate good producers Y_t, Y_t^*, $C_{H,t}$, $C_{F,t}$, $C_{H,t}^*$, $C_{F,t}^*$, $I_{H,t}$, $I_{F,t}$, $I_{H,t}^*$, $I_{F,t}^*$, NX_t satisfying (i) the optimality conditions of each agent; (ii) the optimality conditions of final and intermediate good producing firms; (iii) all markets clear; and (iv) the aggregate resource constraints of both countries.

[8] By Walras' Law, the aggregate resource constraint of the Foreign country is redundant.

[9] Under the law of one price, the terms of trade T, is defined as: $T_t \equiv \frac{S_t P_{F,t}^*}{P_{H,t}} = \frac{P_{F,t}}{P_{H,t}}$.

[10] Thus, our measure of net exports is unaffected by fluctuations in relative prices.

3 Numerical Solution and Calibration

3.1 The Solution Method

To solve the model, we log-linearize the equilibrium conditions around a symmetric deterministic steady state where bond holdings are zero and the steady-state terms of trade is equal to 1.[11] Letting a variable $\widehat{X}_t$ denotes the percentage deviation of X_t with respect to its steady state value $\overline{X}$, the linearized system yields an eight-dimensional system of difference equations:

$$\begin{bmatrix} \Gamma_{t+1} \\ \Lambda_{t+1} \end{bmatrix} = J \begin{bmatrix} \Gamma_t \\ \Lambda_t \end{bmatrix} + V \begin{bmatrix} \varepsilon^z_{t+1} \\ \upsilon_{t+1} \end{bmatrix}, \tag{27}$$

where $\Gamma_t = \left[\widehat{K}_t, \widehat{K}^*_t, \widetilde{B}_{H,t-1}, \widehat{Z}_t, \widehat{Z}^*_t\right]'$ is the state vector, $\Lambda_t = \left[\widehat{C}_t, \widehat{C}^*_t, \widehat{T}_t\right]$ is the co-state vector, ε^z is a (2×1) vector of technological shocks and υ is a (1×1) vector of the sunspot or belief shock.[12] The stability of the dynamic system (27) is determined by the number of eigenvalues of the coefficient matrix J that lie inside the unit circle. If marginal cost is assumed to be decreasing in output (i.e. $\alpha + \gamma > 1$), then the system (27) may not have a unique solution. With this additional returns to scale, the coefficient matrix J can have more eigenvalues inside the unit circle than the number of predetermined variables ($\widehat{K}_t$, $\widehat{K}^*_t$, $\widetilde{B}_{H,t-1}$, $\widehat{Z}_t$, $\widehat{Z}^*_t$), and consequently, multiple solutions to (27) exist.

3.2 Parameterization

The baseline parameter values used to compute the equilibrium are summarized in Table 1. As is standard in the literature, we set the time interval to be a quarter, the discount factor $\beta = 0.99$ and the steady state depreciation rate $\overline{\delta} = 0.025$ (which implies that $\eta \simeq 1.4$). The labor share in production is set equal to 0.7 and the inverse elasticity of labor supply is set $\nu = 0$ (i.e. indivisible labor) to help generate indeterminacy for a small degree of returns to scale; a standard assumption of the indeterminacy literature (see, for example, Benhabib and Farmer, 1994, 1996). The preference parameter ψ is set so that in the steady state the

[11] In the steady state the degree of increasing returns to scale can be expressed as the ratio between average and marginal costs, which in the steady-state is equal to markup: i.e. $\frac{(\alpha+\gamma)(\overline{Y}+\phi)}{\overline{Y}} = \chi$. Consequently, for a steady state to exist, the steady state markup cannot be lower than the degree of diminishing marginal cost i.e. $\chi \equiv \frac{\kappa}{\kappa-1} \geq \alpha + \gamma$.

[12] The variable $\widetilde{B}_{H,t}$ denotes the level deviation of bonds issued by the Home country from its steady state value, relative to steady-state Home consumption.

agent in each country allocates one-third of their time to market activities. In the existing literature, the risk aversion parameter typically chosen is $1 \leq \sigma \leq 2$. Following Stockman and Tesar (1995), we set $\sigma = 2$. In line with Benigno and Thoenissen (2008) we set the bond adjustment cost $\omega = 0.001$ and the steady-state terms of trade equal to 1. We set $a = b = 0.88$ to ensure that the ratio of imports to GDP is equal to 0.12, consistent with the US economy.

Empirical studies offer no clear conclusion on the magnitude of the trade price elasticities, θ and ρ. As discussed by Corsetti et al. (2008a) estimates range from 0.1 to 2. As in Stockman and Tesar (1995) we initially set $\theta = \rho = 1$ broadly consistent with the empirical estimate of Heathcote and Perri (2002).[13] However, the robustness of the numerical results are examined for variations in these parameters. Specifically, following Backus et al. (1994, 1995) and Karabarbounis (2014) we consider a higher parameterization, by setting $\theta = \rho = 1.5$. A low trade elasticity parameterization is also considered, where we set $\theta = \rho = 0.5$.[14] This value is roughly consistent with the estimates of Anderton et al. (2004) and Corsetti et al. (2008a).

A key issue is to generate equilibrium indeterminacy with empirically plausible values for the steady state markup χ. Since intermediate firms only use capital and labor in the production process (12), this implies that the markup is value added. As discussed by Jaimovich (2007), for the U.S. economy value added markups are estimated to lie between 1.2 to 1.4. We set the steady state markup $\chi = 1.2$, consistent with the lower range of the empirical estimates.[15] For the determinacy model we assume that marginal costs are constant (i.e. $\alpha + \gamma = 1$). In this case, with $\chi = 1.2$, $\phi > 0$, and there are zero average profits. For the indeterminacy model we assume that marginal costs are declining (i.e. $\alpha + \gamma > 1$). The numerical analysis suggests that under the baseline parameterization there are many values of α and γ that generate indeterminacy for empirically plausible values of the steady state markup. For simplicity, we follow Hornstein (1993) and set $\alpha + \gamma = \chi = 1.2$, which implies that profits are not only zero on average but also in every period.

[13] Heathcote and Perri (2002) estimate the trade price elasticity for the U.S. to be approximately 0.9.

[14] This value for the trade elasticity is not low enough that the model generates a negative international transmission of productivity shocks á la Corsetti et al. (2008a).

[15] A sensitivity analysis was conducted under a higher value for the steady state markup $\chi = 1.3$, with little significant change in the results found.

Table 1: Baseline parameter values

β	0.99	Discount factor
$\overline{\delta}$	0.025	Steady state depreciation rate of capital
ν	0	Inverse elasticity of labor supply
σ	2	Inverse of the intertemporal substitution elasticity of consumption
θ	1	Elasticity of substitution between home & foreign consumption goods
ρ	1	Elasticity of substitution between home & foreign investment goods
a	0.88	Home bias in consumption goods
b	0.88	Home bias in investment goods
ω	0.001	Bond adjustment cost
$\overline{L}$	1/3	Steady state hours worked
S_L	0.7	Labor share in production
α	0.3 or 0.36	Elasticity of output with respect to capital
γ	0.7 or 0.84	Elasticity of output with respect to labor
χ	1.2	Steady state markup

3.3 Shock Processes

We calibrate the volatility of the shocks so as to minimize the distance between selected model moments and data moments.[16] The objective function is computed as the sum of the squared differences between HP-filtered model moments and data moments, with the identity matrix as the weighting matrix. We substitute the covariance matrix of the shocks that minimizes the objective function with the closest positive semi-definite matrix. Model moments are computed using frequency domain techniques as described in Uhlig (1999), and the estimated sample moments for the data are taken from Gao et al. (2014) and Karabarbounis (2014).[17]

The i.i.d sunspot or belief shock υ_t is introduced into the intertemporal Euler equation of the Home country. As is standard, the log of technology in both countries is assumed to follow an AR(1) process with zero mean. In line with the IRBC literature, we assume that the productivity shocks are quite persistent by setting the autocorrelation parameters $\zeta = \zeta^* = 0.96$. The standard deviations of the productivity shocks used in the baseline parameterization are reported in Table 2. In every alternative parameterization we keep ζ and ζ^* unchanged and we recalibrate the standard deviations of the shocks so as to minimize

[16] It is important to stress that for the moments we do not try to match we generate good results.

[17] Except for the standard deviation and correlation with output of real net exports, which we compute using the Quarterly National Accounts of the OECD.

Table 2: Baseline Shock Processes

	Determinacy model	Indeterminacy model	
		Sunspot shocks only	Sunspot & Prod. shocks
Technology shocks (ε_t, ε_t^*)			
s.d. of Home	0.3168	-	0.2945
s.d. of Foreign	0.3168	-	0.2256
autocorrelations	0.96	-	0.96
corr(ε_t, ε_t^*)	0.3151	-	-0.8030
Sunspot shocks (υ_t)			
s.d.	-	0.9172	0.8992
corr(ε_t, υ_t)	-	-	-0.6804
corr(ε_t^*, υ_t)	-	-	0.1097

the distance between the chosen model and data moments.

Our choice of moments is as follows. In the determinacy model we calibrate the standard deviations and the correlation of productivity shocks so as to match the standard deviation of US output and the correlation between US and Foreign output. In the indeterminacy model with only sunspot shocks we calibrate the standard deviation of sunspot shocks so as to match the standard deviation of US output, which is common practice in the indeterminacy literature. However, when both technology and belief shocks are simultaneously present, as discussed by Donaldson et al. (2013), there is no obvious way to estimate the individual variances of each shocks or their correlation due to the lack of reliable empirical evidence. Consequently, we treat all standard deviations and correlations between the different shocks as free parameters and investigate how far indeterminacy can go in explaining the international macro puzzles.[18] Specifically, in the indeterminacy model the covariance matrix now has six free parameters: the standard deviations of the three shocks and the three cross-correlations between the shocks. The objective function is computed from the following eight moments: the standard deviations of output, the terms of trade and net exports, the correlations with output of the terms of trade and net exports, the correlation of the real exchange rate with relative consumption, and the cross-country correlations of

[18] In this case, sunspot shocks can be correlated with fundamentals - they are no longer pure belief shocks - and the covariance matrix between technology and sunspot shocks can be interpreted as the coordination mechanism to revise expectations.

output and consumption.

4 Results

4.1 The Determinacy Model and the International Macro Puzzles

We start by presenting the main quantitative findings of the determinacy model, which will act as the benchmark for comparison. It is important to stress that with constant marginal costs and fixed capacity utilization the dynamics of the imperfect competition model behave very similar to standard IRBC models.[19] In terms of the steady state, the output-capital ratio and consumption-output ratio are the same. The only difference relates to levels, where steady-state output is lower because of the presence of monopoly power, and thus steady-state capital is also lower. In terms of the linearized model, the only difference between the two model economies comes via the aggregate production technology condition:

$$\widehat{Y}_t = \chi\alpha\widehat{K}_t + \chi\gamma\widehat{L}_t + \chi\widehat{Z}_t, \tag{28}$$

where under imperfect competition $\chi > 1$. Thus output fluctuations generated by productivity shocks are amplified under imperfect competition.

Using the parameter values summarized in Table 1, the unconditional second moments of each model are generated and compared against their empirical counterparts. Columns 2 and 3 of Table 3 report the estimated moments of Hodrick-Prescott filtered variables computed in Gao et al. (2014) and Karabarbounis (2014) respectively, using quarterly data for the period 1973(1) − 2007(4), where the U.S. is taken as the home country.[20] The moments for real net exports are additionally estimated from the authors' own calculations. Columns 4 − 8 of Table 3 report the predicted statistics of the determinacy model under different assumptions for the trade price elasticities and capacity utilization. In the baseline simulation, we assume a fixed capacity utilization rate with unitary values for the trade elasticities $\theta = \rho = 1$ (*Det. baseline*). In the second simulation, we vary the trade elasticity

[19] In the standard IRBC model $\chi = \alpha + \gamma = 1$, given the absence of monopolistic competition.

[20] All series are logged, except real net exports, and Hodrick-Prescott filtered with a smoothing parameter of 1600. The statistics in Gao et al. (2014) are computed where the foreign country is the aggregate of Canada, Japan, and 19 European countries. In Karabarbounis (2014) the statistics are computed using Australia, Canada, Japan, Mexico, Korea, and 12 European countries.

parameters by either assuming $\theta = \rho = 0.5$ or $\theta = \rho = 1.5$. In the third simulation, variable capacity utilization (*var. cap. utiliz.*) is introduced into the baseline model. First, we consider the implications of variable capacity utilization maintaining the assumption of constant marginal costs (CMC), so that increasing returns to scale arise only because of the fixed production cost. Second, we consider the implications when the model exhibits declining marginal costs but indeterminacy does not arise. For this case (DMC) we assume that $\alpha + \gamma = 1.099$, which is sufficiently small given the steady state markup $\chi = 1.2$ to ensure equilibrium determinacy.

Comparison of columns 2 and 3 with column 4 of Table 3 shows that the determinacy model suffers from the same well-established discrepancies with the data for international relative prices and quantities as standard IRBC models. While the data suggests that both the terms of trade and real net exports are counter-cyclical, the determinacy baseline counter-factually predicts that net exports and the terms of trade are both pro-cyclical. Furthermore, a volatility puzzle arises where the predicted volatilities (relative to output) generated by the baseline model are significantly lower than the data. Simulated volatilities for real net exports (0.05), the real exchange rate (0.45) and the terms of trade (0.59) are all much smaller in comparison with the data. It is important to stress that the baseline model can generate sufficient volatility for consumption and employment. This is an important improvement from standard IRBC models and is due to the choice of GHH preferences.[21] Finally, the baseline model suffers from the so-called Backus-Smith puzzle, where the model predict a high positive correlation between relative consumption and the real exchange rate (0.97), whereas in the data this correlation is negative for most OECD countries. By inspection of Table 3, the Backus-Smith puzzle arises for all variants of the determinacy model.

By inspection of the final two columns of Table 3, the introduction of variable capacity utilization has a significant effect on the cross-correlation between net exports and output. This is because, after a positive technology shock, firms increase the utilization rate of capital and the response of output is magnified, particularly in the Home country. Because of higher output, spending on consumption and investment increases. Strong demand for

[21] The ability of GHH preferences to generate higher consumption volatility arises because of the absence of an income effect on labor supply (see equation (16)). Consequently, output changes generate a stronger response of both employment and consumption. For further discussion see Raffo (2008).

Table 3: Second moments of the determinacy model

	Data†		Det.	Variations on the Det. baseline			
				Trade elasticity§		Var. cap. utiliz.	
	Gao	Kar	baseline	$\theta = 0.5$	$\theta = 1.5$	CMC	DMC⋆
Standard deviations‡							
Consumption	0.62	0.83	0.91	0.86	0.93	0.86	0.81
Investment	2.92	2.84	1.38	1.48	1.44	1.83	1.97
Employment	0.68	0.93	0.80	0.75	0.82	0.83	0.91
Terms of Trade	1.77	1.71	0.59	2.11	0.27	0.36	0.21
Real Exchange Rate	2.38	-	0.45	1.60	0.21	0.27	0.16
Real Net Exports	0.38*	0.38*	0.05	0.22	0.09	0.09	0.15
First-order autocorrelations							
Output	0.87*	0.87*	0.72	0.72	0.72	0.71	0.46
Real Exchange rate	0.82*	0.82*	0.75	0.73	0.81	0.74	0.77
Real Net Exports	0.85*	0.85*	0.95	0.75	0.78	0.74	0.70
Correlations with output							
Consumption	0.82	0.86	0.99	0.94	1.00	1.00	1.00
Investment	0.94	0.95	0.96	0.94	0.95	0.96	0.97
Employment	0.85	0.87	1.00	0.95	1.00	1.00	1.00
Terms of Trade	-0.16	-0.17	0.45	0.45	0.42	0.44	0.41
Real Net Exports	-0.47*	-0.45	0.20	0.44	-0.31	-0.41	-0.44
Cross-country correlations							
Output	0.58	0.37	0.58	0.58	0.58	0.58	0.58
Consumption	0.43	0.24	0.77	0.97	0.70	0.71	0.60
Investment	0.41	0.25	0.07	-0.06	-0.01	0.05	0.15
Employment	0.45	0.32	0.70	0.93	0.63	0.64	0.61
Correlation with the real exchange rate							
Rel. Consumption	-0.17	-0.19	0.97	0.90	0.91	0.96	0.89

† The estimated sample moments for the data are taken from Gao et al. (2014) and Karabarbounis (2014) except for values denoted by $*$ which are from the authors own calculations.
‡ The standard deviations of all variables are divided by the standard deviation of output, except for the standard deviation of real net exports which is expressed in absolute terms.
⋆ In the presence of declining marginal costs (DMC) we set $\alpha + \gamma = 1.099$.
§ For all variations in the trade price elasticities we set $\theta = \rho$.

Foreign investment goods ensures that real net exports become negative after a positive technology shock, thus the cross-correlation between net exports and output is negative.[22]

[22] We found that as long as the investment Home bias parameter b is greater than or equal to 0.77, net exports are always counter-cyclical in the variable capacity utilization model (correlation equal to -0.20 or lower), while the choice of elasticities θ and ρ does not change the sign of the correlation between net exports and output.

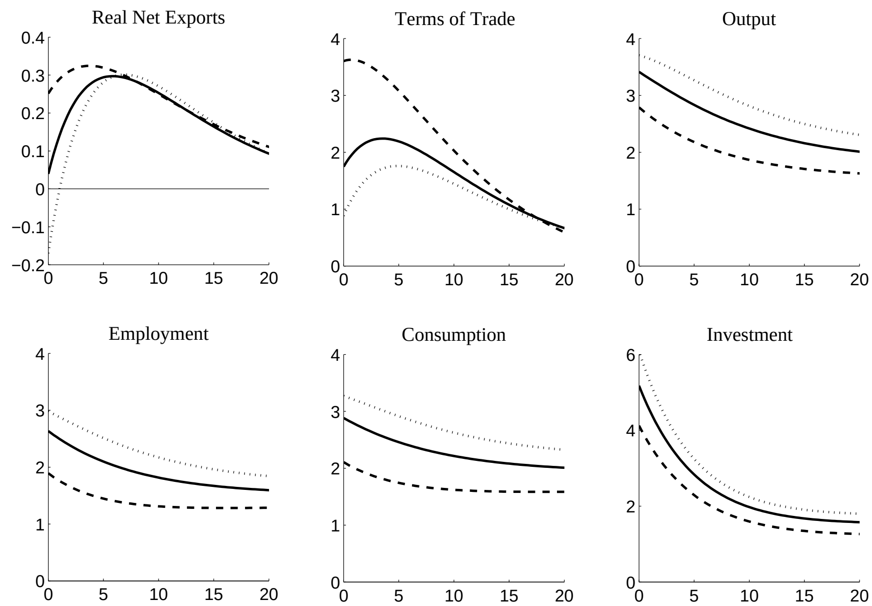

Figure 1: The Determinacy Model - Selected Home impulse responses for a 1% positive Home productivity shock: baseline (—); high trade elasticity (· · ·); low trade elasticity (- - -). Vertical axes: % deviation from the steady state; Horizontal axes: years.

The performance of the model can be improved by choosing a lower value for the trade elasticity parameter ($\theta = \rho = 0.5$). From column 5 of Table 3, this more than triples the volatilities of the terms of trade and the real exchange rate, and more than quadruples the volatility of real net exports relative to the determinacy baseline. Yet, despite these improvements, the model still generates less than 60 percent of the volatility observed for real net exports, and less than 70 percent of the volatility observed for the real exchange rate. By setting $\theta = \rho = 1.5$ (column 6 of Table 3), the model can generate counter-cyclical net exports (-0.31) almost matching the data. However, this is at the cost of further reducing the volatility of relative prices relative to the baseline.[23] Therefore, similar to conventional IRBC models, the determinacy model faces an unpleasant trade-off. Relatively high trade elasticities can be selected to help generate counter-cyclical net exports, or relatively low trade elasticities can be chosen to help improve the volatilities of net exports and relative prices.

To understand this trade-off, Figure 1 reports selected impulse response functions of

[23] As shown by the final two columns of Table 3, allowing for variable capacity utilization has a similar effect.

the home country after a 1% positive productivity shock. In each panel of Fig. 1 the impulse responses are plotted under three alternative values for the trade price elasticity $\theta = \rho = 0.5, 1.0, 1.5$. By inspection, the trade elasticity parameter crucially affects the response of the terms of trade after a productivity shock. If this parameter is relatively low, home and foreign goods are less substitutable for one another. Consequently, a positive productivity shock results in a large deterioration in the terms of trade (i.e. a fall in the relative price of home-produced goods) and a lower increase in domestic output. Hence, the lower the trade elasticities, the higher the volatility of relative prices and the lower the volatility of output in response to productivity changes. Exports rise more than imports, and real net exports, in contrast to the data, are consequently pro-cyclical. With higher trade elasticities, productivity shocks will have a lower impact on relative prices and a higher impact on output, thereby generating counter-cyclical real net exports. Therefore, in order to match the volatility of relative prices *and* the correlation between real net exports and output, determinacy models require a negative international transmission mechanism which only arises by choosing trade elasticities in a very narrow range.[24]

4.2 The Indeterminacy Model

We now consider the indeterminacy model and present our main findings. For the parameter values given in Table 1, columns 5, 6, and 7 of Table 4 summarize the predicted statistics for the indeterminacy model given different assumptions for the trade elasticity parameter. In the first simulation (*Indet. baseline*), we generate indeterminacy employing unitary values for the trade price elasticities $\theta = \rho = 1$. In the other two simulations, we either assume $\theta = \rho = 0.5$ or $\theta = \rho = 1.24$.[25] Column 4 of Table 4 reports the results for the indeterminacy model in the absence of intrinsic shocks.

By inspection of Table 4, the indeterminacy model is able to account for the international relative price and quantity puzzles. First, it can simultaneously generate high volatilities for the terms of trade and real net exports. Indeed, under a lower value for the trade elasticity parameter the indeterminacy model generates volatilities for the terms of trade

[24] For example, as shown by Thoenissen (2010) the range of values for the trade elasticities $\theta = \rho$ that generate this negative transmission in a standard IRBC model is $0.4113 \leq \theta \leq 0.4678$.

[25] We set $\theta = \rho = 1.24$ as this is the highest value for the trade price elasticities that generate indeterminacy with $\chi = 1.2$.

Table 4: Second moments of the indeterminacy model: sunspot shocks and productivity shocks

	Data		Sunspot	Indet.	Variations on Indet. baseline: Trade elasticity§	
	Gao	Kar	shocks only	baseline	$\theta = 0.5$	$\theta = 1.24$‡
Standard deviations						
Consumption	0.62	0.83	0.81	0.91	0.91	0.90
Investment	2.92	2.84	2.24	2.40	1.87	2.83
Employment	0.68	0.93	0.91	1.02	1.01	1.02
Terms of Trade	1.77	1.71	0.73	1.68	1.77	1.63
Real Exchange Rate	2.38	-	0.56	1.27	1.34	1.24
Real Net Exports	0.38	0.38	0.17	0.66	0.38	0.80
First-order autocorrelations						
Output	0.87	0.87	0.73	0.78	0.75	0.79
Real exchange rate	0.87	0.82	0.74	0.71	0.71	0.70
Real Net Exports	0.85	0.85	0.73	0.71	0.72	0.71
Correlations with output						
Consumption	0.82	0.86	1.00	0.99	0.99	0.99
Investment	0.94	0.95	1.00	0.86	0.94	0.80
Employment	0.85	0.87	1.00	0.99	0.99	0.99
Terms of Trade	-0.16	-0.17	0.99	-0.22	-0.17	-0.20
Real Net Exports	-0.47	-0.45	-0.97	-0.41	-0.46	-0.41
Cross-country correlations						
Output	0.58	0.37	-1.00	0.49	0.50	0.36
Consumption	0.43	0.24	-1.00	0.50	0.50	0.38
Investment	0.41	0.25	-1.00	-0.44	-0.16	-0.67
Employment	0.45	0.32	-1.00	0.50	0.50	0.38
Correlation with the real exchange rate						
Rel. Consumption	-0.17	-0.19	0.99	-0.13	-0.19	-0.04

§ For all variations in the trade price elasticities we set $\theta = \rho$.
‡ We set $\theta = \rho = 1.24$ as this is the highest value for which indeterminacy is possible.

(1.77) and real net exports (0.38) that exactly match their empirical estimates.[26] Second, the indeterminacy model correctly predicts that real net exports and the terms of trade are both countercyclical. By inspection of the final two columns of Table 4, these predictions are

[26] Unsurprisingly, the indeterminacy model still generates insufficient volatility for the real exchange rate relative to the data, given the assumption of the law of one price and the absence of non-traded goods. See Corsetti et al. (2008*a*) for further discussion.

robust to alternative calibrations for the trade elasticities. Third, in terms of the correlation between the real exchange rate and relative consumption, the indeterminacy model predicts that this correlation is negative, thereby resolving the Backus-Smith puzzle. As shown by columns 6 and 7 of Table 4, this negative correlation remains regardless of the choice of trade elasticities. Finally, it is important to note that while the indeterminacy model performs well using a combination of extrinsic and intrinsic shocks, the model performs poorly in the absence of productivity shocks. By inspection of column 4 of Table 4, in the presence of only sunspot shocks the indeterminacy model is now unable to resolve any of the major empirical irregularities with the data.

4.3 Solving the Puzzles

4.3.1 The transmission mechanism of sunspot and productivity shocks

To understand these results, Figure 3 depicts the impulse response functions after a 1 percent positive sunspot or belief shock, whereas Figures 4 and 5 depict the impulse response functions after a 1 percent positive productivity shock in the Home country, and Foreign country, respectively. An important element in understanding how sunspot and productivity shocks are transmitted in the indeterminacy model relates to the labor market. The log-linearized Home and Foreign labor demands can be expressed as:

$$\widehat{w}_t = \left[\frac{\alpha(\eta-1)}{\eta-\alpha}\right]\widehat{K}_t + \left[\frac{\eta\gamma}{\eta-\alpha}-1\right]\widehat{L}_t - \left[\frac{(1-a)\eta}{\eta-\alpha}\right]\widehat{T}_t + \left[\frac{\eta}{\eta-\alpha}\right]\widehat{Z}_t, \tag{29}$$

$$\widehat{w}^*_t = \left[\frac{\alpha(\eta-1)}{\eta-\alpha}\right]\widehat{K}^*_t + \left[\frac{\eta\gamma}{\eta-\alpha}-1\right]\widehat{L}^*_t + \left[\frac{(1-a)\eta}{\eta-\alpha}\right]\widehat{T}_t + \left[\frac{\eta}{\eta-\alpha}\right]\widehat{Z}^*_t, \tag{30}$$

where in our parameterization $\eta-\alpha>0$ and $\frac{\eta\gamma}{\eta-\alpha}-1>0$. With decreasing marginal costs, the source of indeterminacy arises from upward-sloping aggregate labor demand schedules which are steeper than the (horizontal) aggregate labor supply schedules.

A positive sunspot shock increases Home consumption directly and Home output increases to satisfy consumption demand.[27] From equation (29), the upward-sloping Home aggregate labor demand schedule shifts down, increasing domestic employment. The sunspot

[27] Higher Home consumption drives up the demand for imports, decreasing net exports. Because of increasing returns, marginal costs are decreasing in output, and as a result, Home goods prices fall and the terms of trade depreciates (increases).

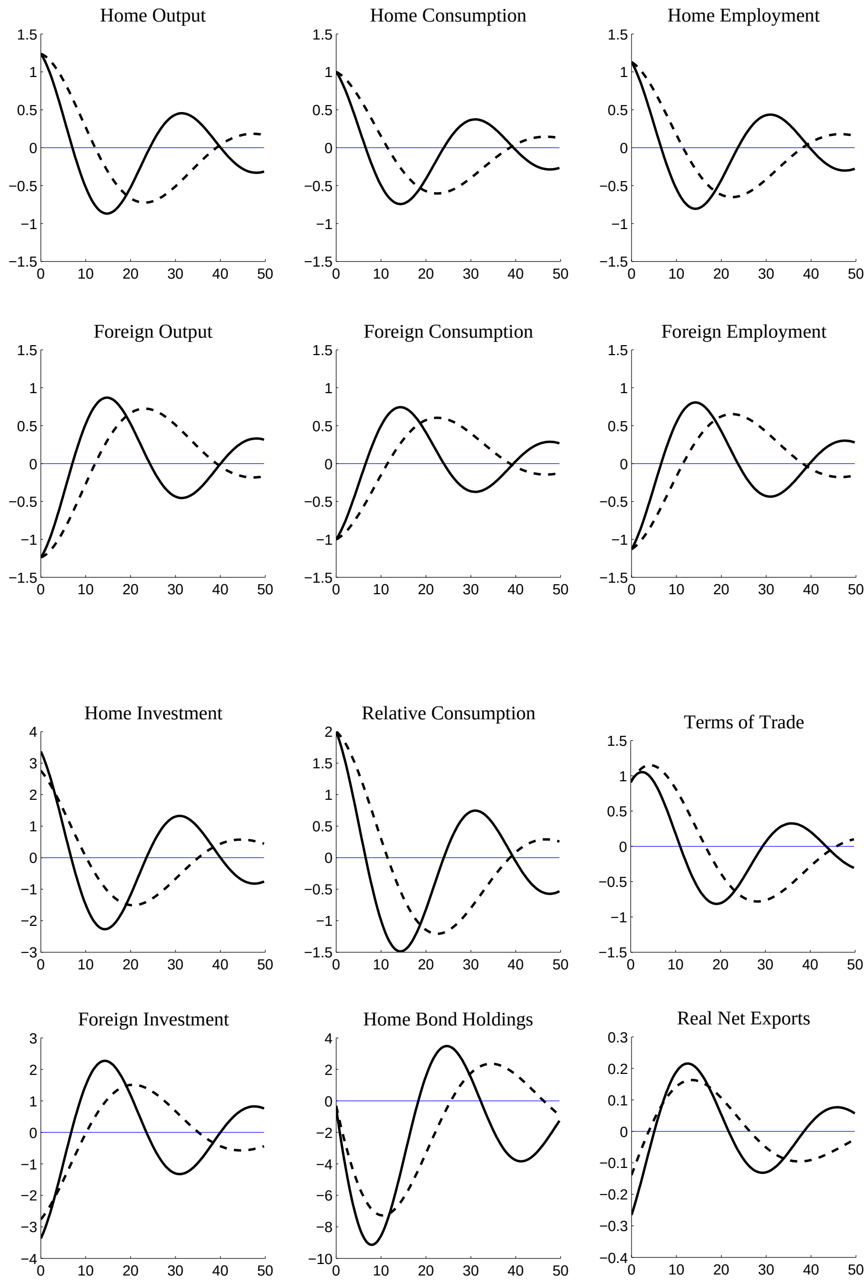

Figure 2: Dynamic responses for the indeterminacy model to a positive 1% sunspot shock: $\theta = \rho = 1$ (- - -) vs. $\theta = \rho = 0.5$ (—). Vertical axes: % deviation from the steady state; Horizontal axes: years.

shock is transmitted to the Foreign country via the terms of trade. From equation (30), the Foreign aggregate labor demand schedule shifts up, and as a result, Foreign employment

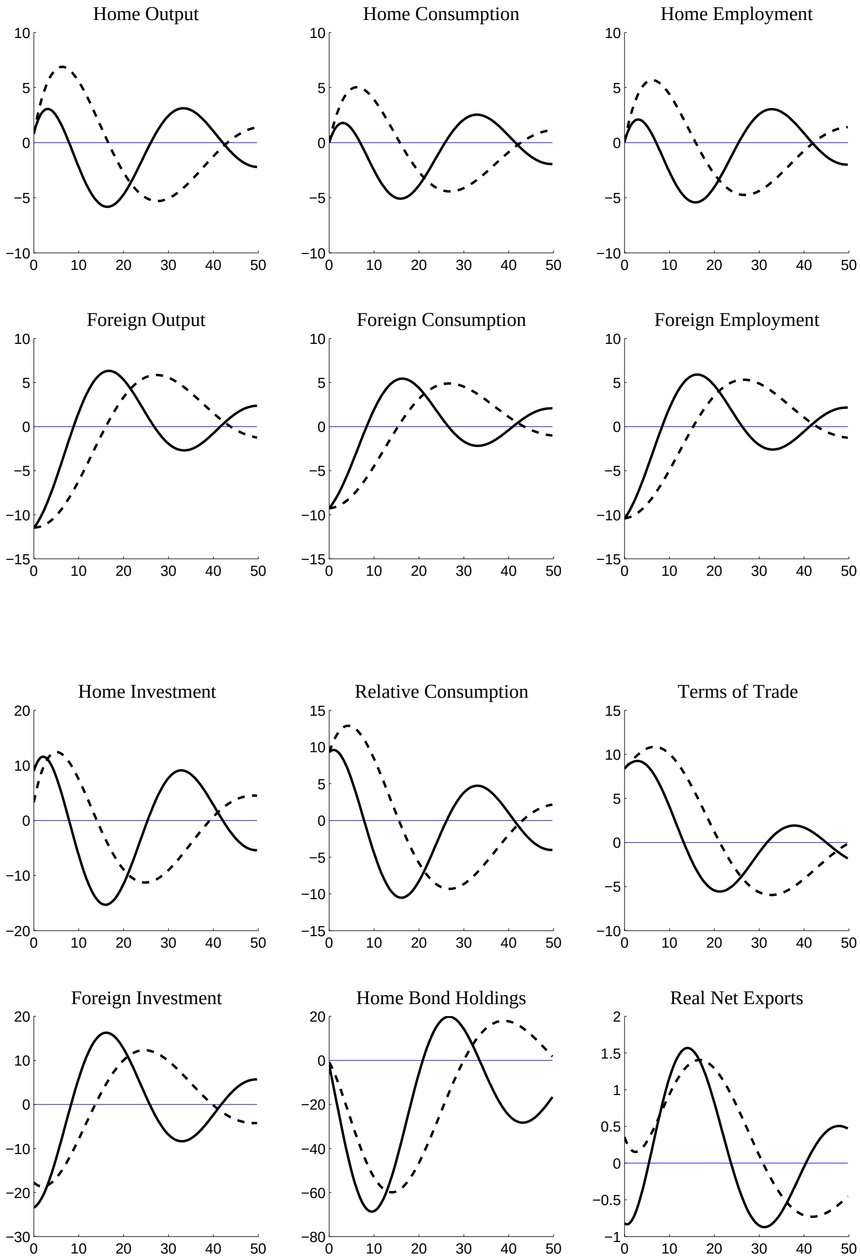

Figure 3: Dynamic responses for the indeterminacy model to a positive 1% Home productivity shock: $\theta = \rho = 1$ (- - -) vs. $\theta = \rho = 0.5$ (—). Vertical axes: % deviation from the steady state; Horizontal axes: years.

decreases, and consequently Foreign output and consumption also decrease. Therefore, absent all other shocks, Home and Foreign correlations are all negative, as shown in Table 3.

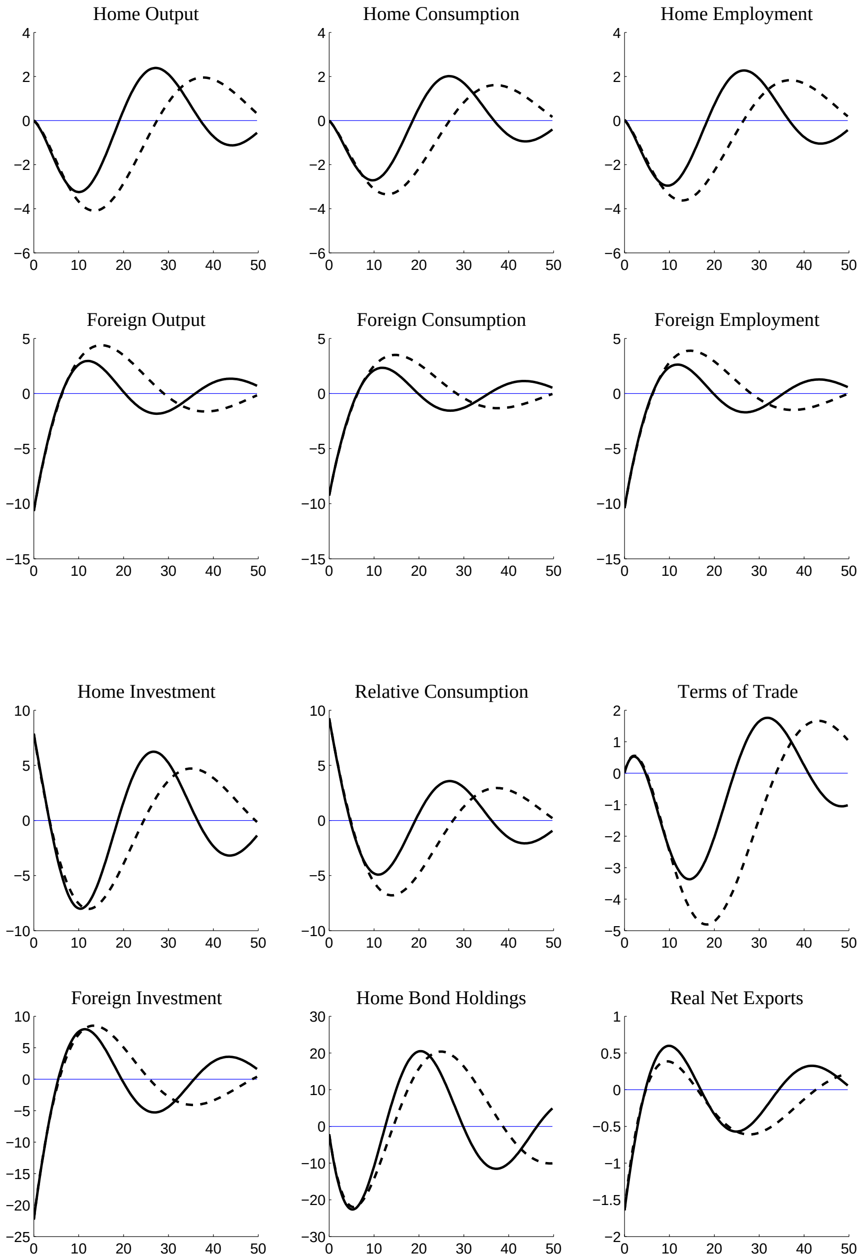

Figure 4: Dynamic responses for the indeterminacy model to a positive 1% Foreign productivity shock: $\theta = \rho = 1$ (- - -) vs. $\theta = \rho = 0.5$ (—). Vertical axes: % deviation from the steady state; Horizontal axes: years.

Consequently, the international transmission mechanism of sunspots shocks is counterfactual, since the data suggests that relative prices appreciate in response to relative output increases. Therefore, the indeterminacy model using sunspot shocks alone cannot replicate the observed behavior for relative prices and quantities or solve the Backus-Smith puzzle.

The ability of the indeterminacy model to replicate the empirical evidence depends on the inclusion of both sunspot and productivity shocks. This arises because of an unusual feature of the indeterminacy model: the propagation mechanism of productivity shocks is asymmetric across the two countries. To see this, first consider the transmission mechanism of a positive Home productivity shock which causes an increase in $\widehat{Z}_t$. As shown in Fig. 3, this results in a depreciation of the terms of trade (as Home marginal costs and Home prices decrease) and an increase in Home investment (as Home investment goods are now relatively more productive). Thus, global demand shifts towards the now relatively cheaper Home goods. By inspection of the first row of Fig. 3, there is a delayed expansion in Home output, consumption, and employment. From equation (29), the shift in labor demand caused by the increase in $\widehat{Z}_t$ is offset by the terms of trade $\widehat{T}_t$ depreciation, such that there is no initial change in employment $\widehat{L}_t$. However, for the Foreign country, from (30) the terms of trade depreciation by shifting the labor demand up reduces Foreign employment $\widehat{L}_t^*$, thus resulting in a decrease in Foreign output and consumption.

Now consider the transmission mechanism of a positive Foreign productivity shock which increases $\widehat{Z}_t^*$. As shown in Fig. 5, there is no initial adjustment in the terms of trade. Instead, an unconventional transmission mechanism is present whereby Foreign quantities initially fall in response to the positive Foreign supply shock.[28] In the absence of a terms of trade adjustment, from equation (30), the increase in $\widehat{Z}_t^*$ results in a reduction in Foreign employment $\widehat{L}_t^*$ and Foreign output initially declines. Thus, goods market clearing is ensured by the contraction in Foreign output (Foreign goods become relative scarcer), rather than via an appreciation of the terms of trade.[29] By inspection of the second row of Fig. 5, this effect is strong but temporary. After the initial fall it takes a period of time for Foreign output to rise above its steady state value.[30]

[28] Without any initial change in the terms of trade, $\widehat{Z}_t^*$ is not transmitted to the Home country.

[29] A positive Home productivity shock implies a decline in the price of domestically produced goods and an increase in the price of the imported good. However, in the case of a positive Foreign productivity shock there is no initial change in relative prices and thus Foreign quantities must fall.

[30] The time it takes for Foreign output to become positive depends on the bond adjustment cost parameter. In our baseline calibration, with a bond adjustment cost of 0.001, Foreign output stays negative for 25 quarters. With a higher bond adjustment cost, e.g. 0.05, the time is reduced to 9 quarters.

4.3.2 The Backus-Smith Puzzle

One important insight of this paper is that the Backus-Smith puzzle can be resolved using a combination of sunspot shocks and the unconventional transmission mechanism for Foreign productivity shocks that arises under indeterminacy. Similar to standard IRBC models, in our indeterminacy model a positive Home productivity shock causes an increase in relative consumption and a depreciation in the terms of trade, implying a positive correlation. However, as we discussed above, the impulse responses of Foreign productivity shocks are neither symmetric nor identical to Home productivity shocks. Consequently, if a positive Home productivity shock ε_t which increases Home technology $\widehat{Z}_t$ is accompanied by a negative Foreign technology shock ε_t^* and a negative sunspot shock υ_t, this can generate a negative correlation between relative consumption and the terms of trade. This happens because the negative shocks ε_t^* and υ_t together strongly counteract the increase in relative consumption generated by the positive shock ε_t, so that the overall effect on relative consumption is negative (i.e. relative consumption falls below the steady state in the short run). However, the impact of the negative shocks ε_t^* and υ_t on the terms of trade is more muted, so that the net effect is still positive (i.e. the terms of trade depreciates above its steady state level in the short-run). Therefore, in our indeterminacy model, the asymmetric responses to Home and Foreign technology shocks, combined with sunspot shocks, can generate a negative correlation between relative consumption and the real exchange rate, provided the variances and correlations of the shocks are chosen with this goal in mind.

4.3.3 The Volatility Puzzles

The indeterminacy model can generate significant improvements in the volatilities of international relative prices and quantities. Compared to the baseline determinacy model, the volatility of the terms of trade and the real exchange rate are over 2.8 times greater and the volatility of real net exports is 13.6 times greater in the baseline indeterminacy model. In traditional IRBC models with home bias, a positive Home technology shock requires a substitution effect towards domestic goods (as they are now relatively more abundant) via a depreciation of the terms of trade. However, in the indeterminacy model the response of the terms of trade (relative to output) is magnified significantly. This happens because now the response of Home consumption after an increase in $\widehat{Z}_t$ is muted, and the response of Foreign consumption is negative. Hence, given the weak demand, (because Home consumption is initially stagnant and Foreign consumption falls), the terms of trade depreciation must be very strong in order to sell the increased supply of Home goods.

4.3.4 The Output Correlation Puzzles

In the indeterminacy model net exports are countercyclical and volatile because of the combined effect of our estimated standard deviations and correlations among shocks. First, notice that a positive Home technology shock has a small but positive effect on net exports. This happens because of the initially muted response of Home consumption, and the terms of trade depreciation (which causes expenditure switching towards Home goods). As a result, a positive Home technology shock causes a decrease in Home imports, which explains the small positive effect on net exports. The positive effect of Home technology shocks on net exports is then magnified by the negative correlation with Foreign technology shocks. This is because a negative Foreign technology shock causes a strong increase in Foreign consumption and consequently a rise in Home exports. This explains why net exports are volatile in our model. To understand why net exports are countercyclical, notice that Home technology shocks are also negatively correlated with sunspot shocks. Whenever a positive Home technology shock occurs at the same time as a negative sunspot shock, the positive effect on net exports of the technology shock is accompanied by the negative effect on Home output of the negative sunspot shock. Thus, the correlation between the shocks explains why net exports are countercyclical.

Before concluding, it is interesting to highlight the role played by the trade price elasticity parameters θ and ρ in the indeterminacy model. By inspection of Table 4, although the quantitative performance of the model is improved by setting $\theta = \rho = 0.5$, in order to solve the puzzles we do not require low trade elasticities as emphasized by the existing literature.[31] In conventional IRBC models, by restricting the trade elasticities within a very narrow range can lead to a downward-sloping world demand for domestic traded goods (with respect to the terms of trade) which generates a negative international transmission for productivity shocks (i.e., the terms of trade appreciate when domestic production expands). The indeterminacy

[31] See, e.g., Heathcote and Perri (2002), Corsetti et al. (2008a, 2008b), Thoenissen (2010).

model can solve the international macro puzzles because of its unconventional features, namely the negative international transmission of Home technology shocks and the negative domestic transmission of Foreign technology shocks (both caused by the upward sloping aggregate labor demand schedule). In contrast to the standard IRBC model, the puzzles can be solved without restricting the trade elasticity.

5 Conclusion

Business cycle statistics suggest that both the terms of trade and real net exports are countercylical and volatile, and the real exchange rate depreciates in response to decreases in relative consumption. Recent studies suggest that the ability of IRBC models to resolve the puzzles crucially rests on some form of negative transmission (see, e.g., Corsetti et al., 2008a.). By allowing for equilibrium indeterminacy, our model can successfully explain these empirical regularities. It has been shown that the empirical success of the model rests with two types of unconventional transmission that arises under indeterminacy: the negative international transmission of Home shocks and the negative domestic transmission of Foreign shocks. In response to supply shocks that originate in the Home country, goods market clearing is achieved via adjustments in the terms of trade. However, for Foreign country supply shocks, goods market clearing occurs via adjustments in Foreign quantities rather than relative prices. Consequently, the model can generate a negative international transmission, whereby the terms of trade depreciate when relative consumption falls, without the need for low values for the trade elasticities.

Given the transmission mechanism uncovered by the analysis, it follows that sunspot or belief shocks alone are not able to explain any of the empirical regularities. Since the ability of our model depends on the inclusion of both sunspot and productivity shocks, we believe this to be a realistic feature of our analysis. Yet there remains a number of anomalies with the data that requires further exploration. Noticeably, our model fails to account for the cross-country correlations observed in the data. The model counterfactual predicts negative cross-country investment correlations and struggles to generate cross-country output correlations higher than cross-country consumption correlations. These issues we leave for future research.

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